

MATLAB-Simulation-Based Analysis of Free RLC Oscillations

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Abstract. Targeting the LC oscillation commonly taught in high school, this study models a series RLC circuit using university-level circuit theory and ordinary differential equations. We derive the analytical solutions for the underdamped, critically damped and overdamped regimes, compare their time responses and energy dissipation, and visualize charge, current, and total energy via MATLAB. The work offers a quantitative, visual reference to bridge high-school and university instruction, and clarifies two often-confused issues: the opposite trends of the Q factor with resistance in series and parallel RLC circuits, and the condition under which critical damping is the fastest route to equilibrium. The main contributions are: (1) a classroom-oriented set of simulation plots for all three damping regimes; (2) a unified formulation based on the damping ratio and on energy dissipation; (3) simple guidance on how R, L and C affect the damping behavior.

Keywords: LC oscillation, RLC model, damping characteristics.

1. Introduction

LC oscillation circuit, with the energy exchange between inductance and capacitance as its core, is an important topic that connects high schools, universities, and even electronic information engineering. Its mathematical form is isomorphic to the harmonic vibration in mechanics, and is often reduced to second-order constant coefficient differential equations under linear quantitative assumptions [1-4]. However, in real situations, LC oscillation circuits are non-ideal and inevitably have resistance, resulting in the temporal variation of current, voltage, and energy being divided into three categories: underdamped, critically damped, and overdamped; energy is no longer conserved, and the frequency of the circuit also changes [5]. In recent years, research on introductory courses and experimental teaching in universities has further clarified several "textbook impressions": for example, the "counterintuitive" phenomenon of the quality factor of parallel RLC increasing with resistance, and new evidence for time optimization problems such as "which damping first returns to the threshold energy" [6-8]; at the same time, numerical studies and teaching practice have shown that simulation is an effective way to connect parameters, waveforms, and energy [9-10]. Based on this, this article takes the common LC circuit in high school as the starting point, and systematically analyzes the analytical solutions and simulated responses of the series RLC circuit, providing a quantitative and visual reference for teaching.

2. Analysis of Basic LC Oscillatory Circuit

In high school physics, the common LC oscillation circuit consists of an ideal inductor L and an ideal capacitor C, as shown in Figure 1. In the initial state, if the capacitor is charged to a voltage U_0 (so that $Q_0 = CU_0$), electrical parameters such as current and voltage will oscillate periodically in the circuit, and energy will transfer periodically between the inductor and capacitor. This process essentially corresponds to an undamped harmonic oscillation, whose dynamic equation can be determined by Kirchhoff's law.

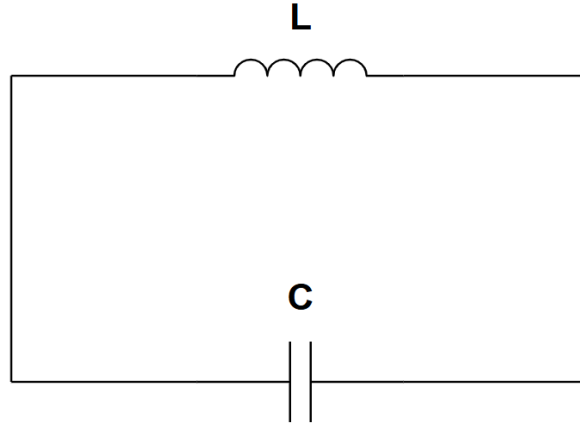


Figure 1. Basic LC oscillation circuit schematic diagram

If the charge on the two plates of a capacitor is $q(t)$ (a periodic function of time in this circuit), then the current is $i(t) = \frac{dq}{dt}$. According to Kirchhoff's voltage law:

$$\frac{q}{C} + L \frac{d^2q}{dt^2} = 0 \quad (1)$$

Equation (1) is a second-order homogeneous linear differential equation; rearranging it gives

$$\frac{d^2q}{dt^2} + \frac{1}{LC} q = 0 \quad (2)$$

Observing equation (2), its equation form is completely consistent with the motion equation of a harmonic oscillator, where the angular frequency is $\omega = \frac{1}{\sqrt{LC}}$.

Therefore, the oscillation period of the circuit is $T = 2\pi\sqrt{LC}$, which is determined only by the size of the inductance and capacitance, and is independent of the initial charge carried by the capacitance. Equation (2) yields the variation of charge over time:

$$q(t) = Q\cos(\omega t + \varphi) \quad (3)$$

It can be seen that Q and φ in equation (3) are determined by the initial conditions. We set the initial charge of the capacitor to Q_0 and the initial current in the circuit to zero, and we can obtain:

$$q(t) = Q_0\cos(\omega t) \quad (4)$$

$$i(t) = -Q_0\omega\sin(\omega t) \quad (5)$$

From this, it can be seen that charge and current have a trigonometric relationship with time, and there is a phase difference of $\frac{\pi}{2}$ between the two, that is, whenever one of them is zero, the other reaches its extremum. In fact, this reflects the mutual conversion between electric field energy and magnetic field energy. We use energy analysis of capacitors and inductors to more intuitively reveal the physical essence of oscillation processes. Substituting equations (4) and (5) into the energy expression for capacitors and inductors, we have

$$W_C(t) = \frac{Q_0^2}{2C} \cos^2(\omega t) \quad (6)$$

$$W_L(t) = \frac{Q_0^2}{2C} \sin^2(\omega t) \quad (7)$$

The sum of the two is exactly $\frac{Q_0^2}{2C}$, constant and unchanging. This indicates that an ideal LC oscillation circuit does not generate energy loss during operation, and energy is only continuously converted between capacitors and inductors: when the capacitor is fully charged, all energy is stored in the form of an electric field; As the charge is released, the current gradually increases, and the magnetic field energy in the inductor accumulates accordingly; When the current reaches its maximum value, energy is completely converted into magnetic field energy. Afterwards, the process proceeds in reverse, and the energy returns to the capacitor, forming periodic oscillations. We used MATLAB to plot the current, charge (Figure 2) and energy (Figure 3) versus time for $L = 10 \text{ mH}$, $C = 1 \text{ } \mu\text{F}$ and $Q_0 = 5 \text{ } \mu\text{C}$ ($U_0 = 5 \text{ V}$), for which the oscillation frequency is $f = 1/(2\pi\sqrt{LC}) \approx 1.59 \text{ kHz}$.

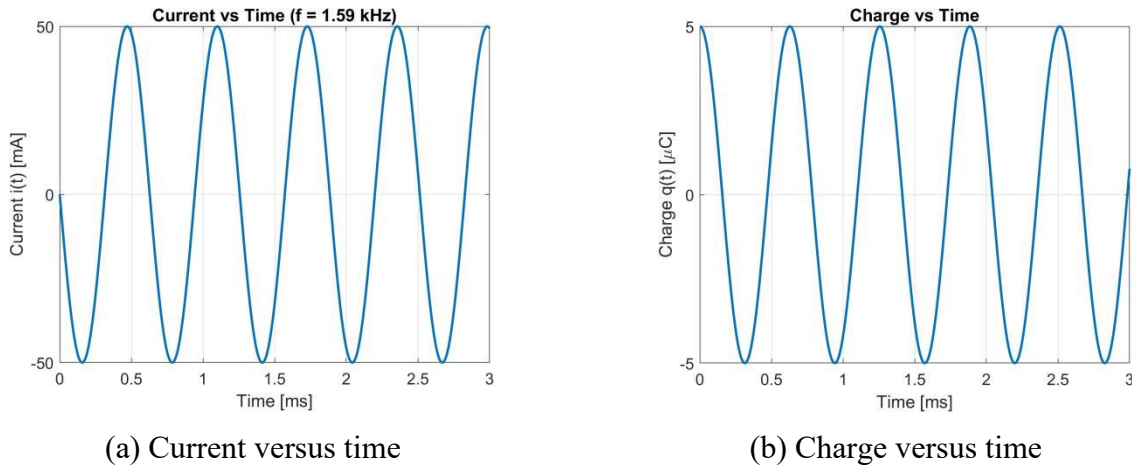


Figure 2 Simulated current and charge versus time in the ideal LC circuit

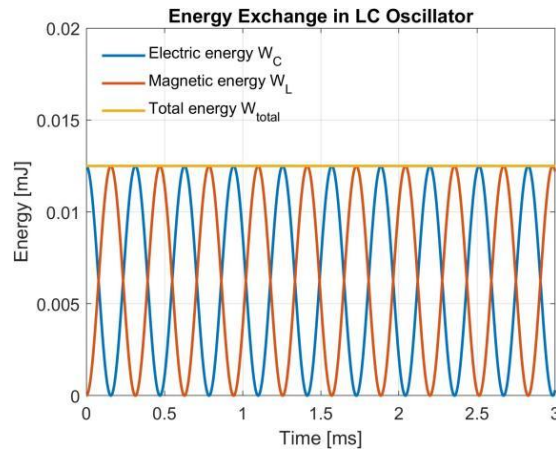


Figure 3 Simulated energy versus time in the ideal LC circuit

In summary, the dynamic essence of basic LC oscillation circuits lies in the fact that the charge satisfies the harmonic oscillation equation, and its period is only determined by the component parameters L and C . Capacitors and inductors serve as "storage tanks" for energy, achieving lossless exchange between electric field energy and magnetic field energy. This process not only reveals the universal laws of electromagnetic oscillation, but also lays a theoretical foundation for further discussions on complex situations involving damping and external power sources.

3. Series RLC Oscillation Circuit

3.1. Modeling and parameter characterization of series RLC free oscillation

When the resistance of the circuit is taken into account, the LC circuit becomes a series RLC circuit consisting of an inductor, a capacitor and a resistor, as shown in Figure 4.

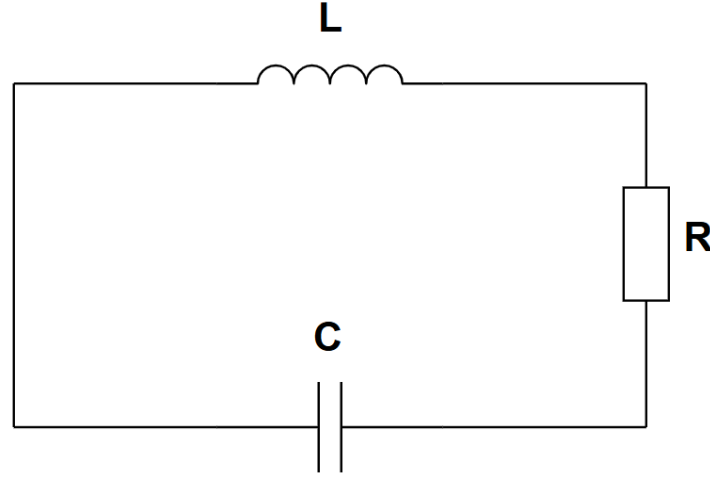


Figure 4 Schematic diagram of the series RLC circuit

According to Kirchhoff's voltage law:

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0 \quad (8)$$

Equation (8) is a second-order linear homogeneous differential equation with constant coefficients, whose structure is isomorphic to that of the damped oscillator in mechanics. For the convenience of physical interpretation and parameterization, we introduce the undamped natural angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ (i.e., the ω of Section 2), the damping coefficient $\alpha = \frac{R}{2L}$ and the dimensionless time $\tau = \omega_0 t$. Equation (8) can then be expressed as

$$\frac{d^2q}{d\tau^2} + 2\zeta \frac{dq}{d\tau} + q = 0 \quad (9)$$

Here $\zeta = \frac{\alpha}{\omega_0} = \frac{R}{2} \sqrt{\frac{C}{L}}$ represents the damping ratio, whose magnitude determines the type of damping behavior. For underdamping $\zeta < 1$, the circuit oscillates with decaying amplitude; for critical damping $\zeta = 1$, the circuit returns to equilibrium without oscillation, and does so fastest among all non-oscillatory cases; for overdamping $\zeta > 1$, the response is a non-oscillatory combination of two decaying exponentials. The most common situation in high school is underdamping, where the amplitude decays exponentially and the oscillation angular frequency $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0 \sqrt{1 - \zeta^2}$ is slightly lower than ω_0 .

To connect the above analysis with the concept of energy, this article defines the instantaneous total energy of capacitors and inductors as:

$$W(t) = \frac{q^2}{2C} + \frac{1}{2} Li^2 \quad (10)$$

From Eqs. (8) and (10), with $U_C = \frac{q}{C}$ and $U_L = L \frac{di}{dt}$ denoting the capacitor and inductor voltages, we obtain:

$$\frac{dW}{dt} = U_C i + U_L i = -i^2 R \leq 0 \quad (11)$$

Equation (11) reveals the essence of damping: in an ideal LC circuit, the electric field energy and magnetic field energy alternate without loss, but with the introduction of resistance, the energy is converted into Joule heat at an instantaneous power of $i^2 R$, so the total energy decreases continuously over time (the rate is zero only at the instants when $i = 0$). For light damping, averaging Eq. (11) over one period gives $\langle \frac{dW}{dt} \rangle \approx -\frac{R}{L} W$, so the energy envelope decays with the time constant $\tau_E = \frac{1}{2\alpha} = \frac{L}{R}$.

From this, a clear tuning guideline follows: for given L and C , increasing R enhances damping, accelerates energy dissipation and lowers the oscillation frequency ω_d . Conversely, reducing R , increasing L or reducing C lowers ζ and helps sustain the oscillation. In engineering this is described by the quality factor $Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{2\zeta}$, which decreases as R increases in a series circuit. For a parallel RLC circuit the trend is reversed, since $Q = R \sqrt{\frac{C}{L}}$ increases with R [6]; mixing up the two configurations is a common source of confusion.

3.2. Analytical solutions and numerical simulations for three damping scenarios

3.2.1. Underdamped situation

When $\zeta < 1$, the characteristic equation of Eq. (8), $s^2 + 2\alpha s + \omega_0^2 = 0$, has a pair of complex conjugate roots $s = -\alpha \pm i\omega_d$. With the same initial conditions as before, i.e., an initial charge Q_0 and zero initial current, the solution is:

$$q(t) = Q_0 e^{-\alpha t} \left(\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t \right) \quad (12)$$

$$i(t) = -Q_0 \frac{\omega_0^2}{\omega_d} e^{-\alpha t} \sin \omega_d t \quad (13)$$

Using the same L , C and Q_0 as in Section 2 with $R = 20 \, \Omega$ ($\zeta = 0.1$), the charge, current and total energy were computed in MATLAB; the results are shown in Figures 5 and 6.

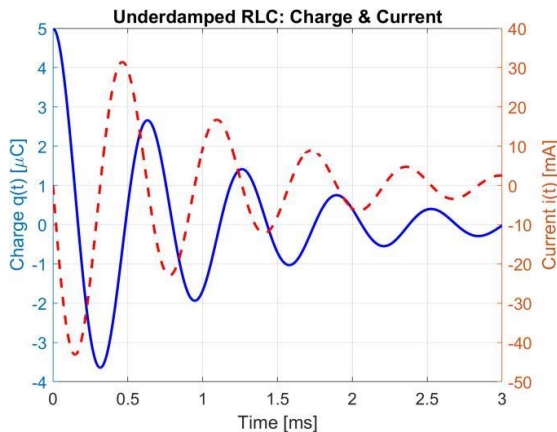


Figure 5 Simulated charge and current versus time in the underdamped case

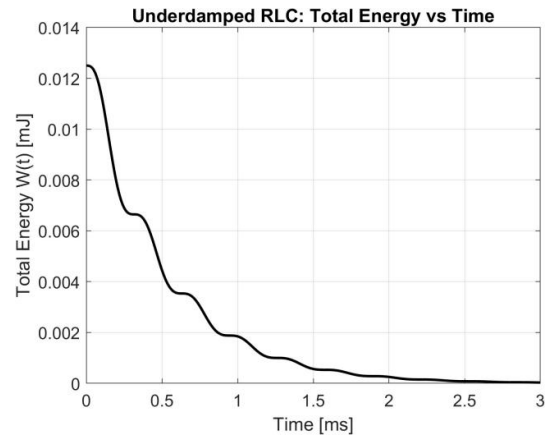


Figure 6 Simulated total energy versus time in the underdamped case

In Figure 5, both traces oscillate with progressively smaller peaks and troughs, and the period is slightly longer than that of the ideal circuit. Since $i = \frac{dq}{dt}$, every zero crossing of the current coincides with an extremum of the charge, whereas the current extrema now occur slightly before the zero crossings of the charge, so their phase difference is no longer exactly $\frac{\pi}{2}$. Figure 6 shows a monotonically non-increasing, staircase-like total-energy curve: the energy falls fastest near the current peaks and forms short plateaus whenever the current passes through zero, with an overall envelope of $e^{-Rt/L}$. Taken together, the two plots display a “strong-to-weak, oscillate-while-fading” pattern and a gradual return toward the baseline.

3.2.2. Critical Damping Scenario

When $\zeta = 1$, the characteristic equation has a double real root $s = -\alpha$ (here $\alpha = \omega_0$), and the solution is:

$$q(t) = Q_0(1 + \alpha t)e^{-\alpha t} \quad (14)$$

$$i(t) = -Q_0 \alpha^2 t e^{-\alpha t} \quad (15)$$

With $R = 200 \, \Omega$ ($\zeta = 1$), the results computed in MATLAB are shown in Figures 7 and 8.

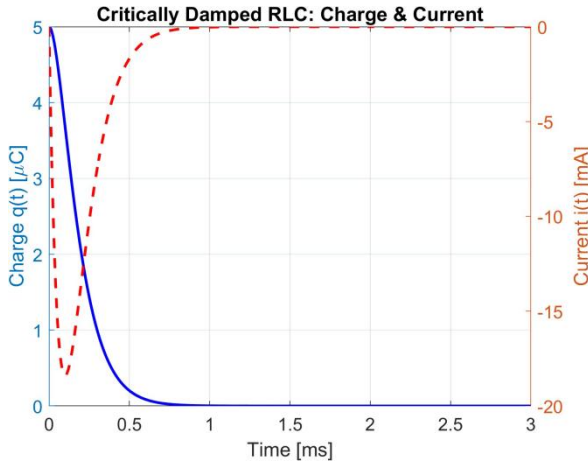


Figure 7 Simulated charge and current versus time in the critically damped case

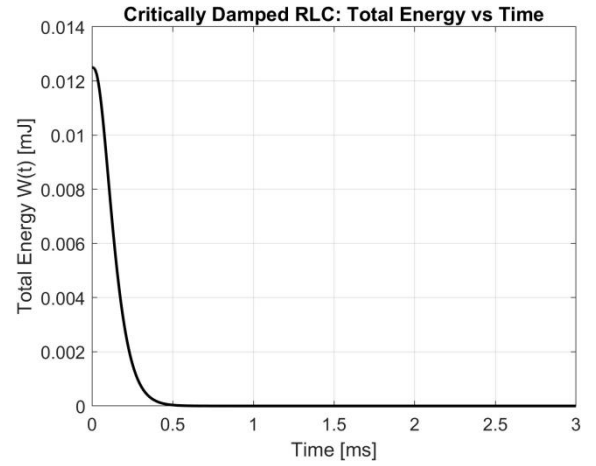


Figure 8 Simulated total energy versus time in the critically damped case

In Figure 7, no oscillation occurs. The charge decreases smoothly and monotonically to zero without overshoot, while the current, starting from zero, shows a single pulse whose magnitude peaks at $t = 1/\alpha$ (0.1 ms here) and then decays without changing sign. Figure 8 shows that the total energy starts with zero slope because $i(0) = 0$, falls most steeply around the current peak and becomes negligible within about 0.5 ms. Note, however, that critical damping is not always the fastest route to equilibrium: if settling is judged by the energy falling below a small threshold, a suitably underdamped circuit can arrive earlier [7]. For the present L and C, the energy drops below 1% of its initial value after about 0.42 ms for $\zeta = 1$, but after only about 0.33 ms for $\zeta \approx 0.7$.

3.2.3. Overdamped situation

When $\zeta > 1$, the characteristic equation has two distinct negative real roots, denoted:

$$s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - \omega_0^2}, \quad (s_1 < s_2 < 0) \quad (16)$$

Solving Eq. (8) with the same initial conditions gives

$$q(t) = Q_0 \cdot \frac{s_1 e^{s_2 t} - s_2 e^{s_1 t}}{s_1 - s_2} \quad (17)$$

$$i(t) = Q_0 \cdot \frac{s_1 s_2}{s_1 - s_2} (e^{s_2 t} - e^{s_1 t}) \quad (18)$$

With $R = 300 \, \Omega$ ($\zeta = 1.5$), the results computed in MATLAB are shown in Figures 9 and 10.

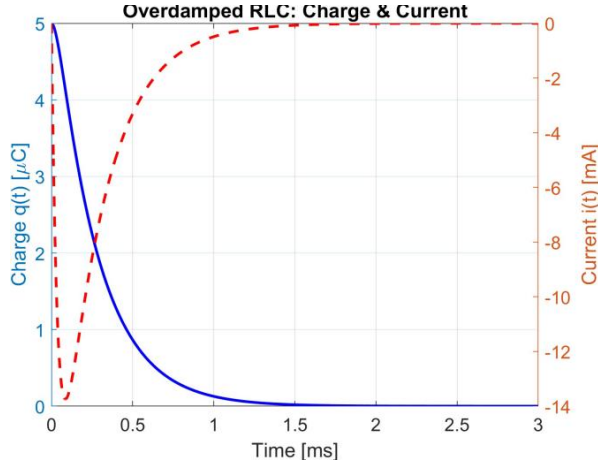


Figure 9 Simulated charge and current versus time in the overdamped case

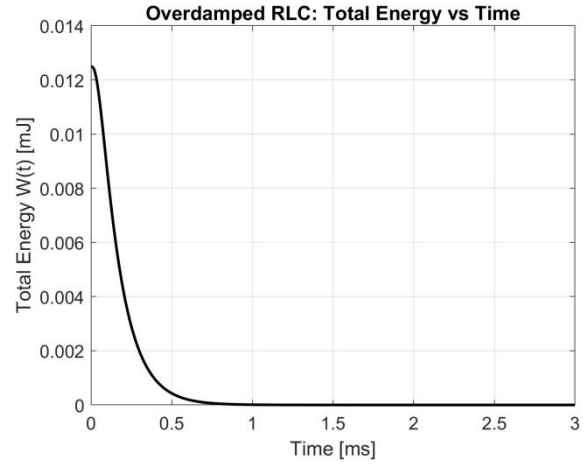


Figure 10 Simulated total energy versus time in the overdamped case

Figure 9 exhibits a non-oscillatory response. The current again starts from zero, reaches a peak of smaller magnitude than in the critical case at about 0.09 ms, and then decays; neither the charge nor the current crosses zero, so there is no overshoot. Compared with Figure 7, the decay shows a pronounced long tail, governed by the slower root s_2 with the time constant $1/|s_2| \approx 0.26$ ms. In Figure 10, the total energy likewise starts with zero slope, falls rapidly around the current peak and then declines slowly; it needs about 0.66 ms to drop below 1% of its initial value, compared with 0.42 ms in the critical case. Together, the plots reveal no oscillation, a longer settling time than critical damping, and a prominent long-tail behavior.

4. Conclusions

This work provides a unified comparison of the time evolution of charge, current, and total energy in a series RLC circuit under underdamped, critically damped, and overdamped conditions, with side-by-side plots that make the contrasts explicit. Starting from the ideal LC circuit taught in high school, the analysis shows how resistance turns lossless energy exchange into monotonic dissipation at the rate $i^2 R$, how the damping ratio ζ determines the type of response, and how R , L and C should be adjusted to sustain or suppress oscillation. It also clarifies that the Q factor decreases with R in a series circuit but increases with R in a parallel circuit, and that critical damping is not always the fastest route to a low-energy state. The present study is limited to ideal components and simulation. Future work will incorporate non-ideal effects, experimental measurements and driven responses, so as to provide a more complete resource for teaching.

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