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# Quantum Tunnelling: Quantum Mechanics Principles and its Applications in Nuclear Physics

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**Abstract.** Quantum physics is a new and popular branch in current physical research. The wave-particle duality and the non-zero probability of showing on the other side of a potential barrier make microscopic particles able to fuse despite it being impossible in classical physics. It plays great significance across many subject areas such as astrophysics, high-energy physics, nuclear physics and even integrated photonics and quantum computing. This paper focuses on extrapolating how quantum tunnelling and the sophistications in quantum systems applies in nuclear physics. The paper does so by using mathematical tools such as WKB approximation, wave function derivations and involves calculations with barrier cross sections. Some counter intuitive observations rise to the surface. For example, results show that nuclei have different shapes and can exchange nucleons prior to tunnelling. In addition, multiple barriers do not affect the mathematical structure of the transmission coefficient, and quantum tunnelling can happen without any barrier present. These arrive at a conclusion that the sophistication of quantum tunnelling needs to be further inquired and all of these contribute more or less to the nuclear reaction rates.

**Keywords:** Wave functions; Gamow Window; Potential barrier; Transmission coefficient.

## 1. Introduction

Quantum Mechanics, when compared to scopes of classical physics such as mechanics, thermodynamics and electromagnetism, is very new and unpredictable. First introduced in the early 20th century by the Copenhagen League, under the context of modelling blackbody radiation, it has thus become the frontier of physics at the time. Previous research has made ground-breaking discoveries of quantum behaviour which cannot be explained by classical physics. These include the photoelectric effect, wave functions, spin-spin duplications, quantum entanglement etc. Nevertheless, the most critical quantum phenomenon, to this paper, is quantum tunnelling, where particles can penetrate a potential barrier that is impossible to climb over in classical mechanics.

This very effect has great significance on the field of nuclear physics, as nuclei can tunnel through the Coulomb potential barrier, formed by repulsion between positive charges, and fall within the potential well to fuse. Despite having $1.5 \times 10^7$K in the core of the Sun, protons are actually in a cold environment where they have insufficient kinetic energy to start fusion in classical physics. Quantum tunnelling, in other words, is the true igniter of the Sun. The review "Quantum Physics of Stars" by M. Wiescher et al. focuses on the Gamow Window, a very narrow range of energy where the chance for tunnelling is high enough for proton-proton fusion [1]. However, it is very immature to only see protons as hard, elastic particles colliding linearly. In the core, plasma state electrons change the shape of the barrier and there are sheer number of protons present. In addition, nuclei are sophisticated quantum systems of which its shape and composing nucleons needs to be considered, especially for very heavy elements [2]. Particles can tunnel through multiple, or zero barriers, and choose a tunnelling pathway with the lowest energy cost. These all have significant impact on the cross-sections, or the probability of tunnelling, which are crucial to understanding high energy physics and to the development of quantum computing just to handle the data.

This research discusses in detail of how quantum mechanics play a critical role in nuclear reactions. The paper will start with fundamental quantum theories and discuss in order the Gamow Window, One-dimensional and Multidimensional Tunnelling, discuss quantum behaviors when tunnelling

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through multiple sequential barriers and share insights into attosecond science with the investigation of tunnelling without barriers. This paper will, in the end, briefly mention the Debye shielding factor and Fermi-Dirac statistics, as well as the applications of this paper's research topic in high-energy physics and quantum computing.

## 2. Method

### 2.1. Wave functions and quantum tunnelling

According to De Broglie Equation, which is demonstrated as

$$\lambda = \frac{\hbar}{p} \tag{1}$$

particles exhibit wave-particle duality. As Heisenberg uncertainty principle suggests, it is not possible to determine position and momentum (motion) of the particles simultaneously. Particles, therefore, have a certain probability of showing up in accordance to the probability distribution function suggested by

$$P(x, x + dx) = |\psi(x, t)|^2 dx \tag{2}$$

The random distribution enables particles to tunnel through potential barriers of which particles do not have sufficient energy to overcome by classical mechanics.

### 2.2. Coulomb barrier and Tunnelling probability functions

The Coulomb potential, from classical electrostatic theorem, is calculated as such:

$$V(r) = \frac{Z1Z2e^2}{4\pi\varepsilon_0} (\text{Coulomb potential}) \tag{3}$$

The environment in the stellar core, despite its tremendously high temperatures, is considered "cold environment" in high-energy physics. This is due to the fact that the Coulomb barrier of protons overcoming their electrostatic repulsion is around 500 keV, yet protons only have 1keV of kinetic energy. Quantum tunnelling enables protons to have non-zero probability of being on the other side of the Coulomb barrier, which enables nuclear fusion.

To be exact, the non-zero probability can be expressed in the following way.$ccP$

$$P \approx e^{-2\int_{r_{min}}^{r_{max}} \sqrt{\frac{2\mu}{\hbar^2}(V(r)-E)}dr} \tag{4}$$

In this expression, $V(r) - E$ represents the height, or the energy of the Coulomb barrier. As the energy of the nuclei, $E$, increases, the height of the Coulomb barrier is reduced. The $\mu$ is calculated from mass of two nuclei by $\mu = \frac{m1m2}{m1+m2}$.

The range of the integral represents the classical forbidden region, in which because potential energy is proportional to $r$, the $r$ makes $V(r) > E$. Classical mechanics forbids the protons from crossing the Coulomb barrier, but wave function allows quantum tunnelling to happen.

From the expression, it is evident that Probability increases exponentially with the energy and mass of the nuclei. By WKB approximation, $P$ can be written as

$$P \approx e^{-2G} \tag{5}$$

where $G$ is the definite integral from nuclear radius $r = 0$ to $r = r_c$ ($r_c$ represents the point where kinetic energy is zero, so $E = V(r_c)$, therefore $r_c = \frac{Z1Z2e^2}{4\pi\varepsilon 0E}$). Thus,

$$G = \frac{1}{\hbar}\int_0^{r_c} \sqrt{2m(V(r) - E)}dr \tag{6}$$

When substituting Coulomb potential $V(r) = \frac{Z1Z2e^2}{4\pi\varepsilon_0}$ into the integral $G$, mathematical simplification obtains

$$G = \frac{\pi Z1Z2e^2}{4\pi\varepsilon 0\hbar}\sqrt{\frac{2m}{E}} \tag{7}$$

By resubstituting $G$ into $P$, it is evident to obtain

$$P \propto e^{-\frac{b}{\sqrt{E}}} \left(b = \frac{\pi\sqrt{2m}Z1Z2e^2}{4\pi\varepsilon 0\hbar}\right) \tag{8}$$

From this simplified expression, when particle energy increases, the probability of tunnelling increases exponentially.

### 2.3. Gamow Peak

Nonetheless, by Boltzmann Distribution, which models the probability of a particle in a given energy state.

$$f(E) \propto e^{-\frac{E}{k_B T}} \tag{9}$$

The function implies otherwise that as energy $E$ increases, the probability of quantum tunnelling decreases exponentially.

The two contradictory distributions suggests that efficient tunnelling requires a specific set of $E$ values. In low energy states, particles are unable to tunnel through the Coulomb barrier according to the tunnelling probability function, however, in high energy states, particles cannot tunnel through either due to the restrictions of the Boltzmann distribution. The energy window is the Gamow Peak, which is directly proportional to $f(E)P(E)$ [3], as shown in Figure 1.

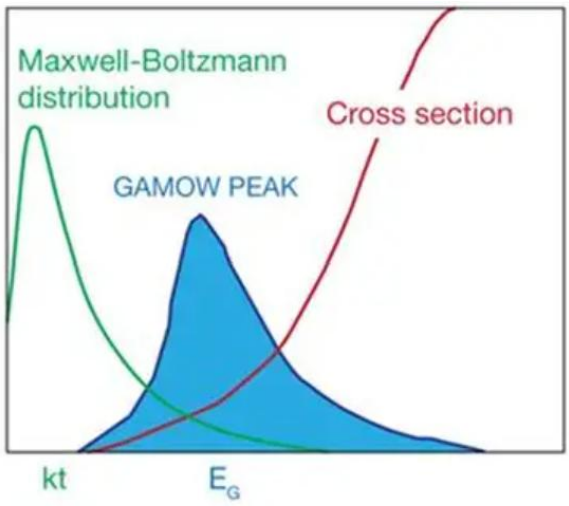


**Fig. 1** Schematic representation of the Gamow Peak [3].

### 2.4. Limitations with One-Dimensional Barrier and Multidimensional Quantum Tunnelling

When this article explains Gamow Peak, it only considers protons as two colinear nuclei approaching each other from infinity. This model, however, suggests rates of $10^2$ order of magnitude lower than reality, which implies that quantum systems in fusion has complexities of which the One-Dimensional Barrier model fails to take account for. The reason is that one-dimensional barrier model still assumes nuclei as elastic balls, which is taking a relatively classical approach.

In studying nuclear fusion below the Coulomb barrier, the experimental observations are the cross section $\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E)$ and the average angular momentum of particles $\langle \ell(E) \rangle = \frac{\Sigma_{\ell=0}^{\infty} \ell \sigma_\ell(E)}{\Sigma_{\ell=0}^{\infty} \sigma_\ell(E)}$. The partial wave cross sections are given by

$$\sigma_\ell(E) = \frac{\pi\hbar^2}{2\mu E}(2\ell + 1)T_\ell(E). [2] \tag{10}$$

From empirical methodologies, the total potential between target and projectile nuclei for the $L^{th}$ partial wave is calculated from

$$V_\ell(r) = V_N(r) + V_C(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} = V_0(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \tag{11}$$

where $V_N$ and $V_C$ are the nuclear and Coulomb potentials [2]. To be exact, $T_\ell(E) = \{1 + \exp[2S_\ell(E)]\}^{-1}$ is the expression for tunnelling probability, whereas $S_\ell(E) = \sqrt{\frac{2\mu}{\hbar^2}} \int_{r_{1\ell}}^{r_{2\ell}} d\,r \left[V_0(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} - E\right]^{1/2}$ is estimated by WKB approximations. The upper and lower limits of the integral are the classical turning points for the $L^{th}$ potential barrier [2].

Assume $V_0(r) = V_{B0} - \frac{1}{2}\mu^2\Omega^2(r - r_0)^2$, where $V_{B0}$ is the height and $\Omega$ is a measure of curvature to the s-wave potential barrier [2]. Using the above method, the transmission probability is calculated to be:

$$T_0(E) = \{1 + \exp[-\frac{2\pi}{\hbar\Omega}(E - V_{B0})]\}^{-1} \tag{12}$$

This is known as Hill-Wheeler Formula. There are significant deviations in low energy conditions because the parabolic assumption no longer holds [2].

In reality, Multidimensional Tunnelling, referred to here as MT, considers nuclei as dynamic quantum systems. This adds several degrees of freedom to nuclear interactions. There are some sophistications mathematical frameworks. One is the coupled-channels formalism, which combines isolated quantum equations together so to monitor the vibration, spinning and nucleon transfer. The other is the barrier distribution method, treating a complex barrier as a distribution of simpler barriers. By differentiating the experimental data obtained twice, one is able to map out the barrier distribution $D(E)$, generated from various nuclear shapes and vibrations;

$$D(E) = \frac{d^2(E\sigma)}{dE^2} \tag{13}$$

Thirdly, there is also a path-Integral and Green's function approaches, which is made widely applicable by Richard Feynman, this methodology considers tunnelling particles are taking all possible paths through imaginary time. It is able to assist in modelling energy dissipation during tunnelling process.

Through these mathematical approaches, one can conclude MT as having the following properties. Nuclei, being quantum systems, has vibration and spins. These quantum properties cause the nucleus to, as described in terms of classical mechanics model, to become ecliptically shaped. These results in nuclei having certain orientations of collision for successful fusion, similar to molecules in chemical reaction kinematics.

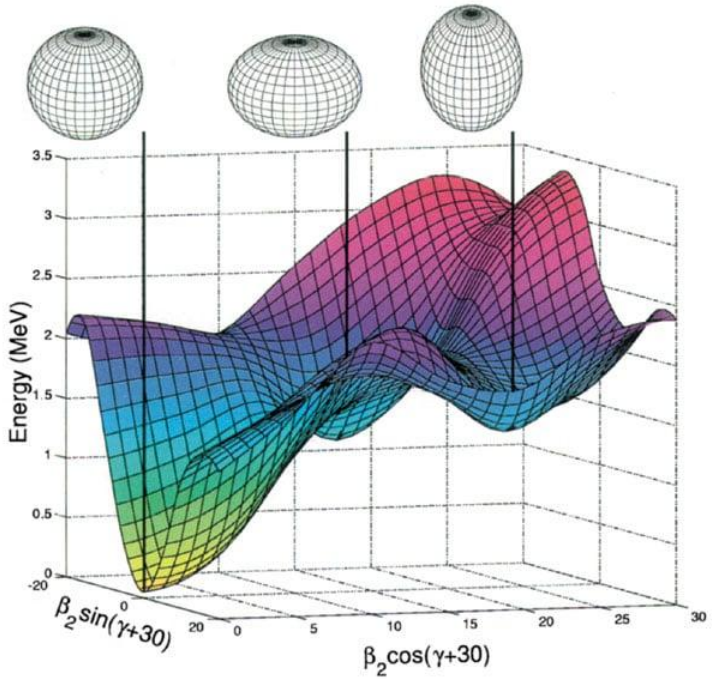


**Fig. 2** Potential energy surface (PES) diagram of a $^{186}Pb$ nucleus at three different shapes [4].

As illustrated by Figure 2, a $^{186}Pb$ nucleus can exist in three different shapes. The spherical nucleus has a near zero potential, while the elongated nuclei have a 1.5 MeV potential [4]. The difference in altitude on the above diagram means that tunneling is not just passing through a wall, but a process of choosing the optimum path on the potential terrain, just as illustrated in Figure 3 [5].

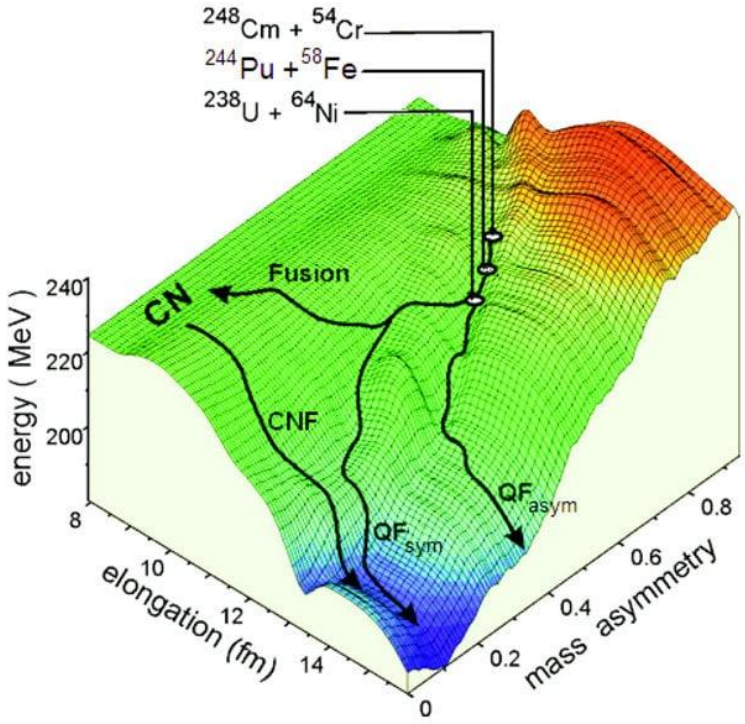


**Fig. 3** Illustration of choosing the optimum path of tunnelling in three different nuclear reactions [5].

In fact, before the actual nuclear interaction, nucleons are able to transfer between nuclei. This affects the rate of fusion by altering the potential barrier. The effect of nucleon transfer on fusion can be illustrated by considering fusion reactions between different $Ni$ isotopes, due to the fact of their varying nucleon numbers. Pioneering experiments lead by Beckerman et al. (1980) first observed the enhancement of sub-barrier fusion across cross sections [6]. Later, Ackermann et al. (1996) measured the cross sections of $^{58}Ni + {}^{64}Ni$ and $^{64}Ni + {}^{64}Ni$ systems and in the following plot energies are normalized to the height of the s-wave potential barrier [6].

As shown in Figure 4, the barrier becomes less energy demanding when $^{58}Ni$ is present instead of $^{64}Ni$. The nucleon exchange narrows and lowers the barrier so that the cross section of tunnelling increases.

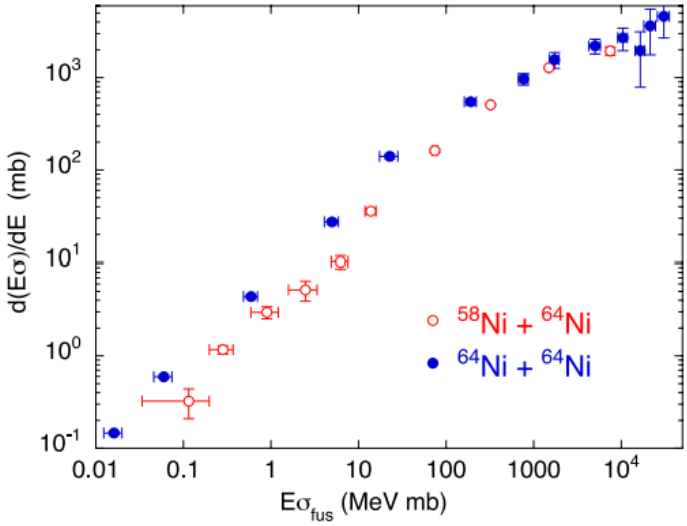


**Fig. 4** Diagram of potential barrier cross section (y-axis) against potential energy (x-axis). There is a left shift in the graph of $^{58}Ni + {}^{64}Ni$ compared with $^{64}Ni + {}^{64}Ni$ [6].

### 2.5. Quantum tunnelling through multiple sequential barriers

Following discussions related to quantum tunnelling with one barrier only, it is worthwhile asking what happens if a second barrier exists after tunneling through the first barrier. In physics, the Transmission Coefficient is a number in range [0,1] that describes the feasibility of a particle successfully tunnelling through a barrier. In the scope of classical probability, the probability of tunnelling through two sequential barriers is calculated using $T(overall) = T(first\ barrier) * T(second\ barrier)$.

This, however, does not work in quantum mechanics, as particles exhibit wave-particle duality. The wave function causes reflection between two barriers, resulting in the observation of interference patterns. In the following derivations using the wave function $(I: x \leq 0,\ II: 0 \leq x \leq a,\ III: a \leq x \leq L,\ IV: L \leq x \leq L + a,\ V: x \geq L + a)$

$$\begin{aligned}
\Psi_I(x) &= A_I e^{\frac{ipx}{\hbar}} + B_I e^{\frac{-ipx}{\hbar}} \\
\Psi_{II}(x) &= A_{II} e^{\frac{-kx}{\hbar}} + B_{II} e^{\frac{kx}{\hbar}} \\
\Psi_{III}(x,t) &= T_1 \left[ A_{III} e^{\frac{ipx}{\hbar}} + B_{III} e^{\frac{-ipx}{\hbar}} \right] \\
\Psi_{IV}(x,t) &= T_1 \left[ A_{IV} e^{\frac{-k(x-L)}{\hbar}} + B_{IV} e^{\frac{k(x-L)}{\hbar}} \right] \\
\Psi_V(x,t) &= T_1 T_2 \left[ A_V e^{\frac{ipx}{\hbar}} + B_{III} e^{\frac{-ipx}{\hbar}} \right]
\end{aligned} \tag{14}$$

In this set of expressions, $AI$, $AIII$, $AV$, $BI$, $BIII$, $BV$ are the transition and reflection amplitudes, and $AII$, $AIII$, $BII$, $BV$ are the coefficients of the evanescent and anti-evanescent waves inside the barrier [7]. It is not this research's primary focus to take a close look at the mathematic reasonings, but in the end it is important to note that the transmission coefficient is calculated as follows:

$$T = \left(16\frac{E}{V}e^{\frac{-2a\sqrt{2mV}}{\hbar^2}}\right)^n \tag{15}$$

The results from the derivation are rather interesting, as the presence of the intermediate between two Coulomb barriers does not affect the mathematical structure of the $T(overall)$. Instead, for n consecutive barriers, the derivation can be applied as well and the $n-1$ spaces do not degrade the transmission coefficient. Despite not being experimentally observed, the theoretical suggestion is already convincing.

### 2.6. Attosecond observations: Quantum tunnelling without a barrier

An attosecond is used to describe an incredibly short time at which a laser pulse is being ejected. The attosecond science is dedicated in investigating the question of whether quantum tunnelling is possible without a Coulomb barrier.

By investigating the effect of intense laser fields on the Coulomb potential, the diagram can be obtained [8]. Interestingly, as seen from Figure 5, when powerful photons are present, the barrier on the right is getting lower. The ionization potential decreases. It is surprising that photons are reducing the barrier, or the cost of electrons escaping its parent atom.

For decades, the physical society believes that this phenomenon happens only when a strong laser creates a thin barrier. However, by shooting a polychromatic laser field, a massive burst of electron tunnelling occurs exactly when the electric field is zero. This is due to the Non-Adiabatic Effects, where instantaneous shift in color and directions in the polychromatic laser field makes the electron wave function impossible to keep up. In addition, as the laser changes frequency, the systematic energy rapidly changes. This makes the electron wave function get caught in a momentum surge, allowing it to escape an atom without the barrier present [9].

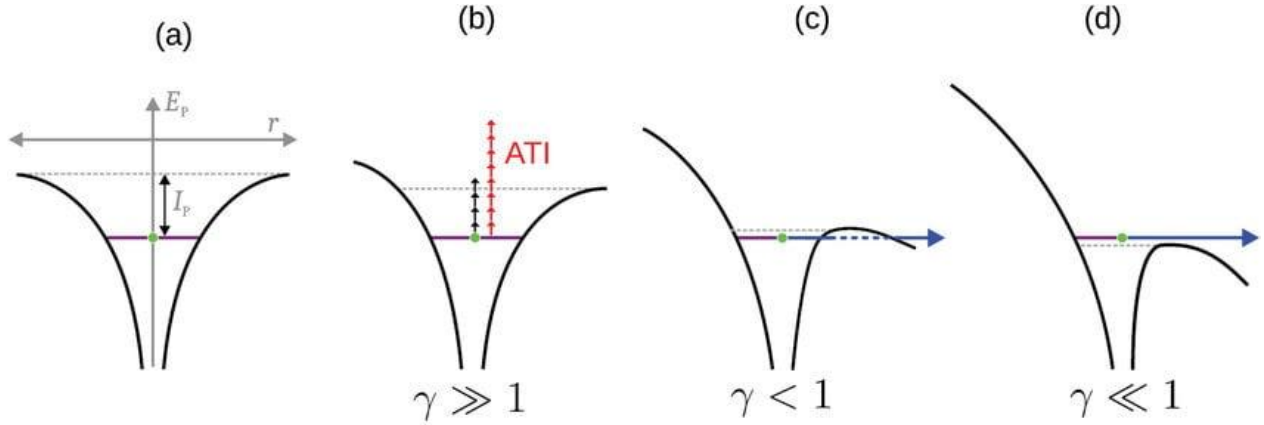


**Fig. 5** The atomic Coulomb potentials under the influence of intense laser fields are shown for the (a) field free, (b) multi-photon ionization, (c) tunnel ionization, and (d) over-the-barrier ionization cases. Above-threshold ionization is also presented as compared to multi-photon ionization (black vertical arrows). Here, $I_p$ is the ionization potential and γ is the Keldysh parameter [8].

## 3. Applications

### 3.1. High energy physics research

In solar environments made of high-energy plasma (including electrons, ions and photons), the Coulomb potential for proton tunnelling is being shielded [10]:

$$V(r) = \left(\frac{Z1Z2e^2}{r}\right) e^{-\frac{r}{\lambda D}} \tag{16}$$

The additional term than before is known as Debye Shielding factor. When the distance r is longer than Debye length. The plasma screening effect greatly lowers the height and the width of the Coulomb barrier. Therefore, the number of protons able to tunnel through per unit time increases, greatly enhancing the solar nuclear fusion rate. In Figure 6, at Debye Length of $\lambda_D = 69.0\mu m$, the screened potential is lower than the unscreened potential.

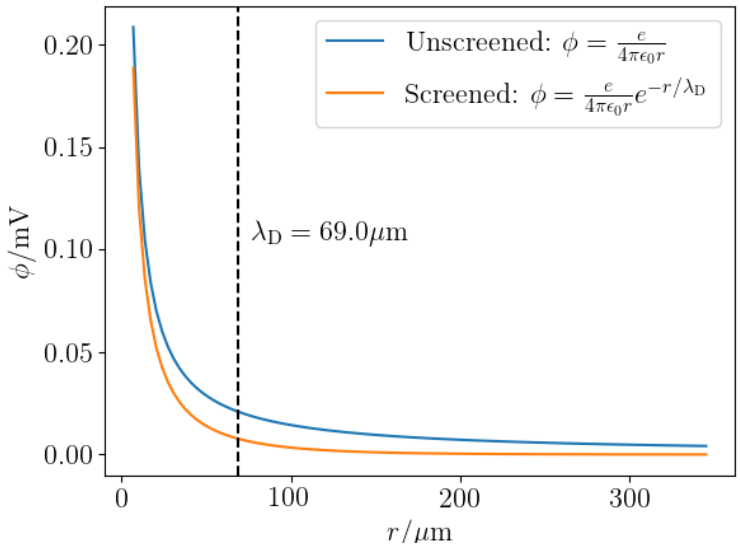


**Fig. 6** Coulomb potential against distance from nuclei plotted by python code [10].

In stellar core environments, it is infeasible to consider protons individually. With stellar core density of $1026\sim1032\frac{protons}{m^3}$, collective quantum behavior, such as Multiphoton ionization, High-harmonic generation, and Quantum Plasmonics, needs to be taken into account.

Even after the main sequence of the sun, the research in quantum mechanics plays a critical role in the physics of the white dwarf. As the core of the Sun has less mass than the Chandrasekhar limit, gravitational force is balanced not by fusion anymore, but by electron degeneracy pressure. To describe it mathematically, this paper introduces the Fermi Dirac statistics (derived from Pauli Exclusion Principle):

$$f(E) = \frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right) + 1} \tag{17}$$

At zero temperature, $f(E)$ is a ladder function consisting of two $f$ values, 0 and 1. It represents at minimum energy, the low-energy orbitals of a fermion are completely occupied while the high-energy orbitals are left vacant. On the contrary, at high temperature scenarios, this statistical function not only becomes continuous, but becomes exactly the graph of Boltzmann distribution

$$f(\varepsilon_i) \approx e^{-\frac{\varepsilon_i - \mu}{k_B T}} = e^{-\frac{E}{k_B T}} \tag{18}$$

By studying fundamental quantum mechanics, high-energy particle physics can be further researched and being applied in fields such as but not limited to nuclear astrophysics and controllable nuclear reactors. In Jinping Underground Nuclear Astrophysics (JUNA) facility, China, high intensity ion beams of $H^+, He^+,$ and $He^{2+}$ are accelerated by a 400-kV electric field. The apparatus is aimed to measure extremely low cross sections of key nuclear reactions with the precision required by astrophysical models [11].

### 3.2. Quantum Computing

To simulate the interaction of quarks and gluons to form protons, neutrons and electrons in the formation of the universe, current physical research relies on mathematical framework classical quantum chromodynamics (QCD) to perform data analysis. However, the theoretical physics research is heavily restricted due to the exponential increase in demand on the computer operations. Henceforth, classical computers create "Sign problem", as the mathematical probability calculations start cancelling terms, creating significant computational errors in dense environments. Classical computers relying on binary and logic gates becomes a bottleneck in human physical research [12].

Quantum computing can actually help researchers understand quantum behaviours more thoroughly, as the natural spin uncertainties and quantum entanglement enables faster computational skills than traditional logic circuits. However, the background interference greatly hinders the accuracy in quantum computations. Furthermore, the number of qubits in present-day computers are modestly small. The current status of this technology is known as the Noisy Intermediate-Scale Quantum (NISQ) era [12]. Physicists are decomposing the problem so that classical supercomputers do the calculations while using quantum computers to deal with the trickiest quantum problems. The progress in physical research therefore, is heavily restricted.

With more advanced quantum research, researchers access more complex quantum computers that enables further broadening of the quantum horizons. The cycle of quantum pushing quantum is essential to both further researching and technological advancements in fields such as Machine Learning and building stronger AI Models.

## 4. Conclusion

In summary, this paper has done investigations into how quantum tunnelling matters to nuclear reactions. It has explained the Gamow Window and fundamental quantum tunnelling theories. The research delves into aspects which adds sophistication to the tunnelling in nuclear reactions, including nuclear orientations, nucleon exchange, shielding, multi-quantum effect, tunnelling through multiple sequential barriers and tunnelling without barriers in attosecond science. The paper has arrived at the conclusion that quantum tunnelling is not the interactions between featureless objects as commonly illustrated, but is the interaction between different quantum systems, which all contributes to the starting and control and nuclear reaction rates. It is the very reason why the Sun can shine for billions of years, instead of millions through gravitational contraction only. The research applies mathematical tools such as WKB approximations, advanced modelling, wave functions derivations, and most importantly, mapping out potential barrier cross sections. The research is heavily based on semi-empirical methods, which uses experimental results from previous research to draw conclusions, instead of relying on first principles in classical physics.

Nevertheless, future research would need to be done. In high energy physics, deepening in understanding of stellar environments is necessary for future inquiries into controllable nuclear reactors, which is a major step forward in making futuristic energy sources applicable. The simulation itself enhances the development of quantum computing, which powers future theoretical physics research and is the foundation to advanced machine learning algorithms. Understanding systems in atomic scale drives technological advancements and broadens the knowledge horizons in the evolution of universe.

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