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Research on the Establishment and Application of Depth Bomb Hit Probability Model Based on Normal Distribution Function

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Abstract. This paper focuses on the anti-submarine technology of depth bomb in complex underwater terrain environments, with a particular emphasis on enhancing the accuracy of hits. Firstly, the anti-submarine warfare process is analyzed, and normal distribution and one-sided truncated normal distribution models are used to simulate the uncertainty of submarine positions. In scenarios where depth errors are negligible, the paper employs the cumulative distribution function to assess the bomb's hit probability, thereby identifying the optimal bombing strategy that achieves a maximum hit probability of 8.69%. This finding is substantiated through the Monte Carlo simulation method. Furthermore, the study extends its analysis to account for the submarine's omnidirectional positioning errors, conducting an independent examination of the hit probability in the depth direction. Based on this, a novel optimal bombing strategy is proposed, which boasts a hit probability of 5.07%, again confirmed by Monte Carlo simulations. Finally, the marginal grid accumulation method is introduced, considering the uniform array arrangement of 9 depth bombs, and the bombing strategy is optimized to increase the hit probability to 29.35%. This study provides a theoretical basis and practical guidance for the optimization of anti-submarine technology using depth bombs, which has important tactical guiding significance for anti-submarine operations.

Keywords: hit probability; normal distribution; Monte Carlo method; cumulative distribution function; marginal grid accumulation.

1. Introduction

In anti-submarine warfare, the process of locating and locking onto submarines is complex. After reconnaissance aircraft initially locate the submarines using advanced electronic equipment, anti-submarine aircraft then carry out the attack mission. The silent evasion strategies of submarines further increase the difficulty of anti-submarine warfare. In recent years, deep-water bomb anti-submarine technology, as an important means of modern anti-submarine warfare, has received extensive attention from scholars both at home and abroad. Relevant research has focused on aspects such as the characteristics of deep-bomb launch, the analysis of hit probability, and the optimization of combat effectiveness. Many achievements have been made: Wan Xiaohui et al.[1]established a calculation model for the vertical launch trajectory of deep bombs, providing theoretical support for launch design; Li Zongji et al.[2] explored the characteristics of self-guided deep bombs in air-sea trajectories, and verified the validity of the hit probability model using the Monte Carlo method; some scholars analyzed the hit probability from perspectives such as firing errors and target parameters, and there were also studies on the distribution law of deep bombs, the continuous firing hit probability, and the optimization of anti-submarine platform combat efficiency[3-10].

However, although the existing research has made significant progress in many fields of depth bomb antisubmarine technology, there are still relatively few studies on establishing accurate mathematical models for the hit probability of depth bomb, predicting and evaluating the hit probability of single depth bomb and multiple depth bombs under different conditions, and optimizing the launch strategy of depth bomb.

This paper focuses on the problem of hit probability in deep-water bomb anti-submarine technology, aiming to improve the striking effectiveness of depth bomb in complex environments through mathematical modeling and simulation analysis. Firstly, we established a coordinate system with the

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submarine's center of gravity projected on the sea surface as the origin and assumed that the three coordinates of the submarine's actual center position are independent of each other, with two following a normal distribution and one following a one-sided truncated normal distribution. Based on this, we proposed the following three research questions:

In the absence of depth errors, analyze the relationship between the maximum bomb hit probability, the plane coordinates of the bomb drop point, and the detonation depth of the depth fuse, and propose a bombing scheme that maximizes the hit probability.

In the presence of positioning errors in all directions of the submarine's center position, derive an expression for the bomb hit probability and explore the detonation depth of the depth fuse that maximizes the hit probability.

For the bombing scheme with multiple depth bomb, using a uniform array arrangement and the same detonation depth for nine depth bomb, calculate the maximum probability scheme for at least one depth bomb to hit the submarine.

To address these issues, we employed the Monte Carlo method and the marginal grid accumulation method, using MATLAB programming to perform exhaustive calculations on possible bomb drop points and detonation depths to find the optimal bombing strategy. This study not only provides a theoretical basis for the optimization of deep-water bomb anti-submarine technology but also offers important references for tactical decision-making in actual combat.

By constructing mathematical models and adopting advanced simulation methods, this research provides a new theoretical perspective and methodology for the hit probability problem of deep-water bomb anti-submarine technology, contributing to the theoretical development in related fields. The research results can provide practical bombing strategies for anti-submarine operations, improving the hit rate and operational efficiency of depth bomb in actual combat, which is of great significance for enhancing the anti-submarine capabilities of naval forces. By optimizing the deep bomb drop scheme, this study helps to improve the tactical use of depth bomb, providing a scientific basis for the design improvement and tactical application of the deep bomb system. On the premise of ensuring combat effectiveness, optimizing the bombing strategy can reduce resource consumption and improve the cost-effectiveness of combat operations.

2. Methods

2.1. Research Approach and Model Construction

To simplify the analysis and focus on the core issues, this study proposes a series of idealized assumptions: the flight time of the depth charge is ignored, the landing point and the detonation depth are precisely controllable; the killing radius of the depth charge is fixed and it descends vertically into the water; a dual fuse system (trigger and depth) is adopted and there are no malfunctions; all depth charges have the same detonation depth. Based on these assumptions, a basic model for the probability of a depth charge hitting the target is established. The symbols are defined as follows: the length, width and height of the submarine are respectively L,D,H, heading angle of the submarine is β, the central coordinate is (X, Y, Z), the impact point of the deep shell is (x, y, h), the killing radius is R.

2.2. Research Tools

2.2.1. Normal Distribution Function

It is used to describe the probability density of the error of the impact point of a deep projectile, where the horizontal coordinates X and Y follow a normal distribution with a mean of 0 and variances of σ_x and σ_y respectively.

2.2.2. Cumulative Distribution Function (CDF)

It is used to calculate the probability of a random variable within a specific interval, and random samples are generated through the inverse CDF for Monte Carlo simulation.

2.2.3. Monte Carlo Method

Based on random sampling to simulate a large number of experiments, it is used to verify the accuracy of the analytical model.

2.2.4. Marginal Grid Accumulation

It is used to solve the multi-dimensional integration problem in the probability calculation of multiple deep projectiles, by constructing a grid and accumulating the probability values of the effective intervals.

3. Results And Discussion

3.1. The Maximum Hit Probability Model When Submarine Depth Positioning is Accurate

3.1.1. Analysis and Establishment of the Model

Assuming that the depth positioning of the submarine is error-free, the horizontal coordinates X and Y follow a normal distribution. There are three possible scenarios for the depth bomb hitting the submarine:

Situation 1 (Detonation by Contact Fuze): The impact point is within the submarine's plane, and the detonation depth $h \geq Z + \frac{H}{2}$ (below the submarine's upper surface).

Situation 2 (Above-determined depth detonation): The impact point is within the submarine's plane, the detonation depth $h < Z + \frac{H}{2}$, and the explosion point is within the kill radius R covering the submarine's upper surface.

Situation 3 (Surrounding-determined depth detonation): The impact point is outside the submarine's plane, but when detonated, the submarine is within the kill radius R.

A comprehensive analysis reveals that when the detonation depth h is located between the upper and lower surfaces of the submarine ($Z - \frac{H}{2} - R \leq h \leq Z + \frac{H}{2} + R$), the effective killing plane range of the deep bomb is the largest, extending r units around the submarine's surface. At this time, the hit probability P is the highest, and the expression is:

$$P = \left[\Phi\left(\frac{x + \frac{L}{2} + R}{\sigma_x}\right) - \Phi\left(\frac{x - \frac{L}{2} - R}{\sigma_x}\right)\right] \cdot \left[\Phi\left(\frac{y + \frac{D}{2} + R}{\sigma_y}\right) - \Phi\left(\frac{y - \frac{D}{2} - R}{\sigma_y}\right)\right]$$

According to the properties of the normal distribution, the probability that the landing coordinates (x, y) are closer to the origin (0, 0) is greater.

3.1.2. Model Solution

For the following parameter values: $\mathrm{L} = 100\mathrm{m}, \mathrm{D} = 20\mathrm{m}, \mathrm{H} = 25\mathrm{m}, \beta = 90°, \mathrm{R} = 20\mathrm{m}, \sigma_x = \sigma_y = 120, \mathrm{Z} = 150\mathrm{m}$.It can be calculated that the optimal bomb-throwing plan is as follows: the coordinate of the landing point is (0,0), and the detonation depth hh is within the range of [162.5, 182.5] . The calculation result of the maximum hit probability is:

$$P_{\max} = [\Phi(0.58) - \Phi(-0.58)] \cdot [\Phi(0.25) - \Phi(-0.25)] \approx 8.69\%$$

The probability contour plots (Fig.1) and the comparison of probabilities at different points (Fig.2) were all drawn using MATLAB, which verified that the probability is the highest when the impact point is at the origin and the detonation depth is appropriate.

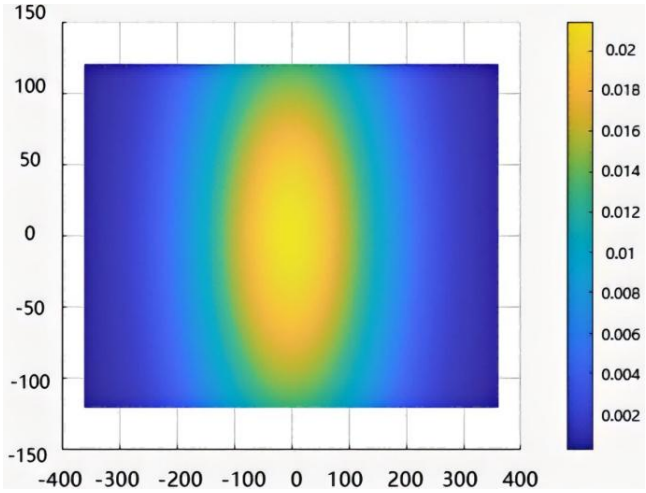

Figure 1. Contour map of probability

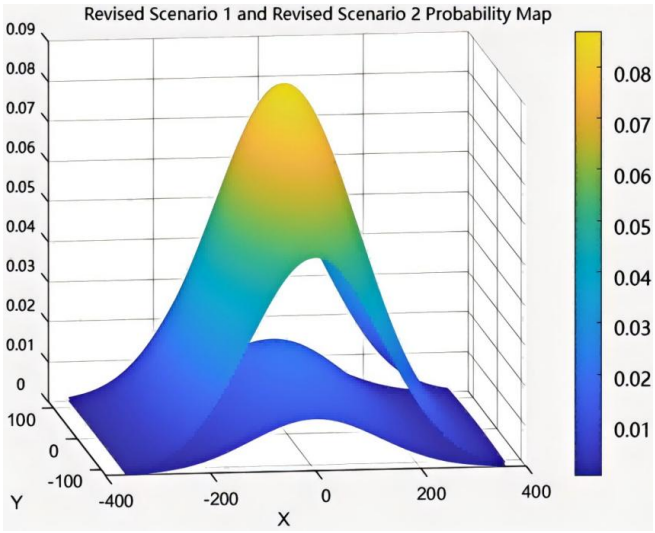

Figure 2. The comparative graph of hit probabilities for two different detonation depths

3.1.3. Model Verification

Using the Monte Carlo method, a sufficient number of random numbers following the $N(0,120^2)$ distribution were generated. Conditional judgments were made based on three hitting scenarios, and the simulation probabilities were calculated. The results were consistent with the model's calculated values, verifying the validity of the model.

3.2. Model for Maximum Hit Probability When Submarine Depth Positioning is Uncertain

3.2.1. Analysis and Establishment of the Model

The three coordinates of the submarine's center position are independent and have positioning errors. $X \sim N(0, {\sigma_x}^2)$, $Y \sim N(0, {\sigma_y}^2)$, and Z follows a one-sided truncated normal distribution $N(h_0, {\sigma_z}^2, l)$, where h_0 is the depth positioning value of the submarine's center position, and l is the minimum actual depth of the submarine's center position. The probability density function of Z can be expressed as:

$$f_{h_0,\sigma_z,l}(v) = \frac{1}{\sigma_z} \cdot \frac{\phi\left(\frac{v-h_0}{\sigma_z}\right)}{1-\Phi\left(\frac{l-h_0}{\sigma_z}\right)} \qquad (l < v < +\infty)$$

The probability of hitting in the depth direction is the probability that the detonation depth h falls within the range $[Z-\frac{H}{2}-R\,, Z+\frac{H}{2}+R]$. By deriving the probability density function of Z and the standardized upper and lower bounds, the calculation formula for the depth direction hitting probability is obtained.

Since depth and planar coordinates are independent, the total hitting probability is the product of the maximum hitting probability when depth positioning is accurate and the depth direction hitting probability.

$$P=\left[\Phi(\frac{x+\frac{L}{2}+R}{\sigma_x})-\Phi(\frac{x-\frac{L}{2}-R}{\sigma_x})\right]\cdot\left[\Phi(\frac{y+\frac{D}{2}+R}{\sigma_y})-\Phi(\frac{y-\frac{D}{2}-R}{\sigma_y})\right]$$

$$\cdot\int_{1}^{+\infty}\left[\Phi(\frac{\left(Z+\frac{H}{2}+R\right)-h_0}{\sigma_z})-\Phi(\frac{\left(Z-\frac{H}{2}-R\right)-h_0}{\sigma_z})\right]f_{h_0,\sigma_z,l}(v)\mathrm{d}v$$

3.2.2. Model Solution

Set parameters: $L=100\mathrm{m}$, $D=20\mathrm{m}$, $H=25\mathrm{m}$, $\beta=90°$, $\sigma_x=\sigma_y=20$, $R=20\mathrm{m}$, $h_0=150\mathrm{m}$, $\sigma_z=40$, $l=120$.The calculated depth hit range is [Z−32.5, Z+32.5].

Using MATLAB, calculate the probability of h being within the range of [87, 300], and draw the corresponding probability density graph (Fig. 3).

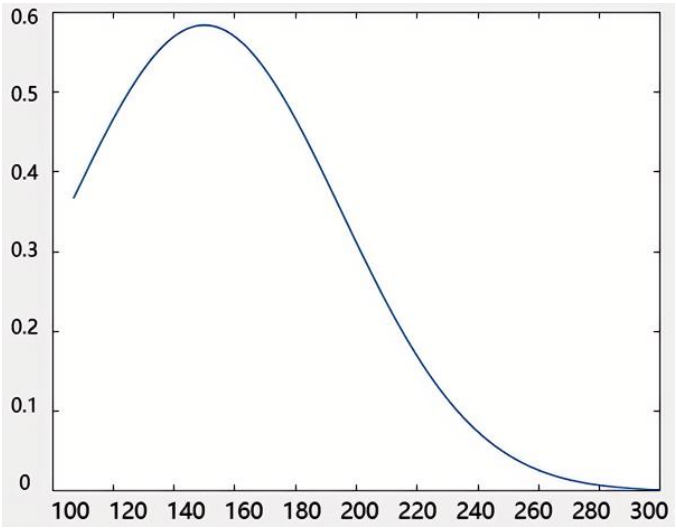

Figure 3. Depth probability variation graph

As shown in the figure, when the detonation depth h = 150, the hit probability in the depth direction of a deep bomb is the highest, with the maximum hit probability reaching 58.35%.

Based on the above results, the total maximum hit probability is 0.0869 × 0.5835 = 0.0507. That is, when there is a positioning error in all directions, the maximum hit probability of a single deep bomb is 5.07%. The optimal solution is to drop at the point (0,0) and set the detonation depth at 150m.

3.2.3. Model Verification

Using the Monte Carlo method, random numbers following the distribution N(150, 40², 120) were generated. The proportion of samples where h falls within the range of 150 ± 32.5 was calculated. The simulated probability was consistent with the model's calculated value, thereby verifying the validity of the model.

3.3. Maximum Hit Probability Scheme for Multiple Deep Bomb Dropped

3.3.1. Model Analysis and Establishment

When dropping 9 deep bombs in a 3×3 matrix pattern, the heading is at an angle of a with the same direction, and at an angle of b in the vertical direction. The coordinates of the 9 depth charges are $(x + ia, y + jb)(i, j \in \{-1,0,1\})$. The objective is to determine the values of h, a and b that will maximize the probability of at least one deep hit.

The marginal grid cumulative method is adopted to solve this problem:

(1) Eliminate invalid accumulation: Ensure that the killing ranges of each deep bomb do not overlap as much as possible to cover more areas with high probability density.
(2) Optimization strategy: Due to the high probability density at the center and low at the edges, the optimal array center should coincide with the submarine positioning center.
(3) Interval selection: Starting from the initial values based on the submarine size and the killing radius (140m, 60m), analyze the impact of the interval changes on the cumulative probability.

3.3.2. Model Solution

Using MATLAB, the optimal intervals a, b and the detonation depth h were searched through a traversal process.

Plane probability: When the intervals a = 140m and b = 60m, and the array center is aligned with the origin, the plane coverage probability is the highest, reaching 50.30%.

Depth probability: The same as 3.2, the optimal detonation depth h = 150m, with a probability of 58.35%.

Total probability: The maximum probability of at least one deep bomb hitting P = 50.30% × 58.35% = 29.35%.

The results show that a reasonable array arrangement can significantly improve the hit probability, from 5.07% for a single bomb to 29.35%.

4. Conclusions

In this paper, the probability model of anti-submarine aviation depth bombs based on normal distribution proposed by us has shown high flexibility and operability. The model provides clear mathematical expressions and calculation steps, making the design and optimization process of bombing schemes easy to operate. Through numerical calculation and simulation, we can conveniently evaluate the effects of different bombing schemes, which has important reference significance for modern anti-submarine operations and is easy to extend to multi-weapon systems and dynamic environments. However, the model also has shortcomings such as simplified assumptions, insufficient risk assessment, parameter sensitivity, and failure to consider dynamic factors. Nevertheless, the model still has a wide range of promotion potential and can be used for multi-target combat analysis, error correction, and tactical training, providing strong theoretical support for anti-submarine operations. In summary, this model provides a powerful tool for the probability analysis of anti-submarine aviation depth bombs. Although it has certain limitations, its flexibility and operability give it a wide range of application prospects in the field of anti-submarine operations. Future research can aim to improve the model's shortcomings to further enhance its accuracy and practicality.

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References

[1] Wan Xiaohui, Peng Shi, Zhang Haibo, et al. Modeling of vertical launch trajectory and analysis of range influence law of Shipborne deep ammunition [J]. Journal of Underwater Unmanned Systems, 2023,31 (06): 926-933.

[2] Li Zongji, Cheng Shanzheng, Liu Yang. Monte Carlo simulation method for calculating the hit probability of aviation homing depth projectile [J]. Journal of Missiles and Guidance, 2012,32 (02): 22-24.

[3] Khaikov ,L. V.Assessment of the single shot hit probability as a function of the horizontal range taking into account different target types and points of aim[J].Vojnotehni č ki glasnik, 2019,67 (1): 13-35.

[4] Ou Yanghuan, Li Zifen, Bi Simin, et al. Hit probability analysis and simulation calculation of tandem aerial homing depth projectiles [J]. Equipment Management and Maintenance, 2017, (02):116-118.

[5] Wang Wei. Research on guidance and control methods of deep-water bombs [D]. China Ship Research Institute, 2015.

[6] Zeng Jiayou, Song Youkai, Yan Jiangang. Calculation model of acquisition probability of anti ship missile based on circular distribution of target position error [J]. Tactical Missile Technology, 2010, (02):55-58+63.

[7] Jiang Xuankai, Zhao Xuetao, Jia Yue. Analysis on hitting probability of aviation homing depth missile attacking submarine [J]. Fire and Command and Control, 2009,34 (08): 64-67.

[8] Lin Xianjie, Jia Yue, Zhao Xuetao, et al. Model and Simulation of submarine attack efficiency of aviation homing depth projectile [J]. Journal of Missiles and Guidance, 2008, (04):131-134.

[9] Chen Zunyin. Simulation Research on hit probability of aerial homing torpedo [J]. Ship Science and Technology, 2002, (S1):7-9.

[10] Li Jingxian. Research on hit probability calculation model of airdrop homing torpedo weapon [J]. Avionics Technology, 2001, (02): 6-8+23.